

# Quiz 1, MATH 240, Fall 2023

Write your name clearly.

Name:

Section Number:

UID:

- (1) [20] **Using elementary row operations**, determine if the system of linear equations has a solution or not. If it does, find an explicit solution. If it does not have a solution, briefly explain why.

$$x_1 - 3x_2 + 4x_3 = -4$$

$$3x_1 - 7x_2 + 7x_3 = -8$$

$$-4x_1 + 6x_2 + 2x_3 = 4$$

$$\left[ \begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & 2 & 4 \end{array} \right] \xrightarrow[R_3 + 4R_1]{R_2 - 3R_1} \left[ \begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 18 & -12 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{6}R_3} \left[ \begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 1 & -3 & 2 \end{array} \right] \xrightarrow[R_2 - 2R_3]{R_1 + 3R_3} \left[ \begin{array}{ccc|c} 1 & 0 & -5 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 2 \end{array} \right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[ \begin{array}{ccc|c} 1 & 0 & -5 & 2 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow[R_2 + 3R_3]{R_1 + 5R_3} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

So the system has a unique solution, given by

$$x_1 = 2$$

$$x_2 = 2$$

$$x_3 = 0$$